Monotonicity in distributivity with binominal *each*

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Introduction Recent studies treat binominal *each* (b-*each*) as a distributive numeral marker, showing that the empirical generalizations from and the theoretical framework for studying distributive numerals can be fruitfully extended to analyzing b-*each* (e.g., Cable 2014, Champollion 2015, Kuhn 2017). On this view, what has been traditionally called the host of b-*each* is actually a numeral expression being marked as a distributive numeral. Building on this line of research, the current study shows that it's not the number word itself that b-*each* cares about, but the measure function inside the host. This argument is supported by the fact that b-*each* requires this measure function to be monotonic in association with distributivity.

To implement this distributivity-related monotonicity condition, a dynamic plural logic is proposed in the tradition of van den Berg (1996) and Nouwen (2003) (DPlL). The proposed account explains a few eccentric properties of b-*each*, including (i) why it can only be hosted by so-called 'counting quantifiers' (Szabolcsi 2010), (ii) why the host must take narrow scope relative to distributivity (Choe 1987), and (iii) why the host seems to be subject to an 'evaluation-level plurality' constraint (Champollion 2015). Due to space limitations only (i) is demonstrated in the abstract.

The proposed account sides with recent studies on distributive numerals that it is fruitful to treat distributivity as a discourse-level plurality, but departs from them in terms of (i) the logical framework for building this plurality (DPlL vs. P(lural)-C(ompositional)-DRT), and (ii) the lexical constraint introduced by b-*each* (and distributive numerals) that constrains this plurality (monotonicity vs. evaluation-level plurality.)

Monotonicity (informally) Numeral expressions are known to host b-*each* (e.g., Safir and Stowell 1988). However, Zhang (2013) observes that not all numeral expressions are equal, as b-*each* is sensitive to the type of measure functions they embody. Extensive measure functions like DISTANCE (1a), VOLUME (1b), and CARDINALITY (13) give rise to friendly hosts; but non-extensive measure functions like SPEED and TEMPERATURE give rise to hostile hosts.

(1) a. The boys walked 3 miles each. DISTANCE
b. The drinks measure 50 ounces each. VOLUME

(2) a. *The boys walked 3 miles per hour each. SPEED
b. *The drinks measure 50 degrees (Fahrenheit) each. TEMPERATURE

Similar contrasts have been found in other measurement constructions like pseudo-partitives and comparatives. Schwarzschild (2006) and Wellwood (2015) capture the contrast by positing the monotonicity condition: a measure function mapping elements in its domain to elements in its range is said to be monotonic if and only if the order in the domain is preserved in the range, as formalized in (3).

(3) A measure function $\mu : D_{\preceq^{Part}} \mapsto D_{\preceq^{Deg}}$ is monotonic iff: (Wellwood 2015)
for all α, β, if $\alpha \prec^{Part} \beta$, then $\mu(\alpha) \prec^{Deg} \mu(\beta)$.

In this paper, I propose a **distributivity-related monotonicity condition**. Informally, in constructions with b-*each*, the monotonicity condition must be checked relative to the part-whole structure of the distributivity key (DistKey). For example, (1a) expresses more than just distributivity, i.e., that each of the boys walked 3 miles. It also expresses that the measurement in the host preserves the part-whole relation present in the DistKey. So, boy1 is part of the plural entity boy1⊕boy2 and the walking distance of boy1 is less than the walking distance of boy1⊕boy2. By contrast, while (2a) also expresses distributivity, i.e., each boy walked 3 miles per hour, it cannot make the same order-preservation claim, as the walking speed of a plurality is not greater than that of its parts (rather, it's an average of the speed of its parts).

Importantly, the monotonicity condition is not (just) a property of the measurement of the host, but a property anchored at distributivity—it must be checked after the host is evaluated distributively. DPlL has two nice features enabling us to model this distributivity-related monotonicity. One, it models distributivity as a discourse-level plurality with dependency information, so the monotonicity condition can be stated using this plurality. Two, its dynamic nature allows us to sequence the checking of monotonicity after the evaluation of distributivity.

DPlLM Extending DPlL (van den Berge 1996, Nouwen 2003) with domain pluralities and measure functions, I propose DPlL with measurement (DPlLM). In DPlLM, interpreting a formula yields a relation between **sets** of assignments G, H, instead of assignments g, h in Dynamic Predicate Logic. The default strategy for introducing variables is defined in (4), which pointwisely manipulates an input set of assignments and does not introduce any dependency between the new variable and extant ones. This is an essential difference between DPlLM and PCDRT.

(4) $G[\![[x]]\!]H$ iff $\exists \mathcal{D}.H = \{g^{x \to d} \mid g \in G, d \in \mathcal{D}\}$ ($\mathcal{D}$ may include atomic or plural entities)

Lexical relations are checked collectively, in the way defined in (5).

(5) $G[\![R(x_1, ..., x_n)]\!]H$ iff $G = H$ and $\left\langle \bigoplus H(x_1), ... \bigoplus H(x_n) \right\rangle \in \Im(R)$, where $H(x) := \{h(x) \mid h \in H\}$

Distributivity is modeled via the distributive operator δ_x, defined in (6), which introduces dependency between variables (formalized via $G|_{x=a}(y)$). δ_x splits up the input set G into subsets based on the values stored in the variable x. It then pointwisely checks that the formula ϕ in its scope holds for each subset.

(6) $G[\![\delta_x(\phi)]\!]H$ iff $G(x) = H(x)$ and $\forall a \in G(x).\ \text{ATOM}(x)\ \&\ G|_{x=a}\ [\![\phi]\!]\ H|_{x=a}$, where
$G|_{x=a}(y) := \{g(y) \mid g \in G, g(x) = a\}$

Monotonicity (formally) I propose that b-*each* selects a measure phrase as its host. It lexically encodes a monotonicity condition on the measure function, as in (7), which says: the measure function μ is monotonic relative to two discourse referents x and y, with x storing the plurality contributed by the DistKey (i.e., the subject), and y storing the values introduced by the host. The monotonicity condition is satisfied iff measuring y's value in a set storing more x's values yields a bigger number than measuring y's values in a set storing fewer x's values. In other words, the part-whole relation embodied by the DistKey is preserved in the measurement of the host.

(7) $G[\![\mathbf{MC}_{x,y}(\mu)]\!]H$ iff $G = H$ and $\forall D, D' \subseteq G(x).D \subset D' \to \mu\left(\bigoplus G|_{x\in D}(y)\right) < \mu\left(\bigoplus G|_{x\in D'}(y)\right)$,
where $G|_{x\in D}(y) := \{g(y) \mid g \in G, g(x) \in D\}$

Compositionally, b-*each* returns a higher order generalized quantifier (GQ) after combining with a measure phrase like *two books*, as in (8). The **MC** test applies to the measure function **cardinality** introduced by *two books*. Conjunction in (8) is dynamic and is defined in (9).

(8) two books each$_{x,y}$ $:= \lambda c.c\left(\lambda P.[y] \wedge \text{BOOK}(y) \wedge \mathbf{card}(y) = 2 \wedge P(y)\right) \wedge \mathbf{MC}_{x,y}(\mathbf{card})$

(9) $G[\![\phi \wedge \psi]\!]H = \exists K.G[\![\phi]\!]K\ \&\ K[\![\psi]\!]H$

The higher order dynamic GQ looks for a function from GQ to truth values, puts a GQ back in the scope of this function (Charlow to appear), and introduces the **MC** test outside the scope of this function, as illustrated in the LF (10). Here, the null operator Dist encodes δ, giving rise to distributivity, and the DistKey *the boys* introduces a maximal discourse referent, defined as in (11).

(10) two books^y each$_{x,y}$ λQ $\big($ the boys^x Dist $\lambda x.(Q\ (\lambda y.\text{READ}(x, y)))$ $\big)$

(11) a. the boys^x $:= \lambda P.\mathbf{max}^x(\text{BOY}(x)) \wedge P(x)$

b. $G[\![\mathbf{max}^x(\phi)]\!]H$ iff $G[\![[x] \wedge \phi]\!]H$ and $\neg\exists H'.H(x) \subset H'(x)\ \&\ G[\![[x] \wedge \phi]\!]H'$

The operation shown in (10) is essentially a 'split scope' mechanism that allows the measurement information on *two books* to scope both inside and outside of distributivity. Scoping it inside distributivity gives us the correct narrow scope reading of *two books* and scoping it outside distributivity allows the monotonicity condition to check the measurement in association with distributivity.

(12) $\mathbf{max}^x\left(\text{BOY}(x)\right) \wedge \delta_x \begin{pmatrix} [y] \wedge \text{BOOK}(y) \wedge |y| = 2 \wedge \\ \text{READ}(x, y) \end{pmatrix} \wedge \mathbf{MC}_{x,y}(\mathbf{card})$

(12) says: for each of the boys, e.g., a, b and c, he read two books and for any subsets D and D' of $\{a, b, c\}$, if D (e.g., $\{a\}$) is subset of D' (e.g., $\{a, b\}$), the cardinality of the books that the boys in D read is smaller than the cardinality of the books that the boys in D' read.

The CQ constraint The proposed analysis directly captures the so-called counting quantifier constraint of b-*each*. Since the **MC** test applies to measure functions, a good host of b-*each* must be able to provide a monotonic measure function. This is what distinguishes good hosts, i.e., counting quantifiers introduced by (modified) numerals, *many* and *few*, from hostile hosts, i.e., definite NPs, bare NPs and regular indefinites introduced by *a* and *some*.

(13) The girls read [$_{\text{host}}$two/more than three/a few/many *{the, those, $\emptyset$, some} books] each.